OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4729**

Mechanics 2

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

4729

INSTRUCTIONS TO CANDIDATES

*   Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully and make sure that you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
*   The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
*   You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets **[ ]** at the end of each question or part question.
*   **You are reminded of the need for clear presentation in your answers.**
*   The total number of marks for this paper is **72**.
*   This document consists of **4** pages. Any blank pages are indicated.

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Turn over

1 Find the average power exerted by a climber of mass 75 kg when climbing a vertical distance of 40 m in 2 minutes. **[3]**

2 A small sphere of mass 0.2 kg is dropped from rest at a height of 3 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.8 m above the ground.

   (i) Calculate the magnitude of the impulse which the ground exerts on the sphere. **[5]**

   (ii) Calculate the coefficient of restitution between the sphere and the ground. **[2]**
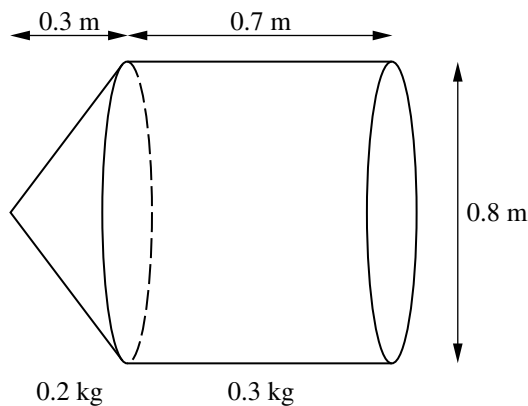
3

0.3 m   0.7 m

0.8 m

0.2 kg   0.3 kg

Fig. 1

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

   (i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. **[4]**
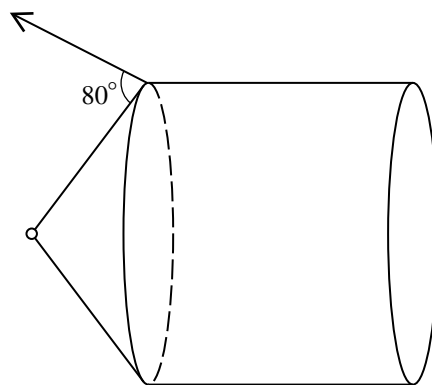

80°

Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

   (ii) Calculate the tension in the wire. **[4]**

4 A car of mass $700\,\text{kg}$ is moving along a horizontal road against a constant resistance to motion of $400\,\text{N}$. At an instant when the car is travelling at $12\,\text{m s}^{-1}$ its acceleration is $0.5\,\text{m s}^{-2}$.

   (i) Find the driving force of the car at this instant. **[2]**

   (ii) Find the power at this instant. **[2]**

The maximum steady speed of the car on a horizontal road is $35\,\text{m s}^{-1}$.

   (iii) Find the maximum power of the car. **[2]**

The car now moves at maximum power against the same resistance up a slope of constant angle $\theta°$ to the horizontal. The maximum steady speed up the slope is $12\,\text{m s}^{-1}$.

   (iv) Find θ. **[4]**

5 Two spheres of the same radius with masses $2\,\text{kg}$ and $3\,\text{kg}$ are moving directly towards each other on a smooth horizontal plane with speeds $8\,\text{m s}^{-1}$ and $4\,\text{m s}^{-1}$ respectively. The spheres collide and the kinetic energy lost is $81\,\text{J}$. Calculate the speed and direction of motion of each sphere after the collision. **[12]**

6
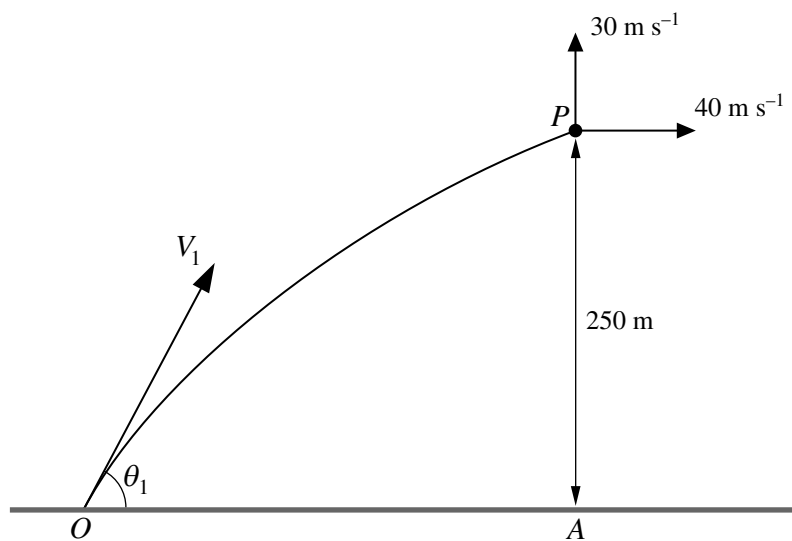
A particle P is projected with speed $V_1\,\text{m s}^{-1}$ at an angle of elevation θ_1 from a point O on horizontal ground. When P is vertically above a point A on the ground its height is $250\,\text{m}$ and its velocity components are $40\,\text{m s}^{-1}$ horizontally and $30\,\text{m s}^{-1}$ vertically upwards (see diagram).

   (i) Show that $V_1 = 86.0$ and $\theta_1 = 62.3°$, correct to 3 significant figures. **[5]**

At the instant when P is vertically above A, a second particle Q is projected from O with speed $V_2\,\text{m s}^{-1}$ at an angle of elevation θ_2. P and Q hit the ground at the same time and at the same place.

   (ii) Calculate the total time of flight of P and the total time of flight of Q. **[4]**

   (iii) Calculate the range of the particles and hence calculate V_2 and θ_2. **[8]**
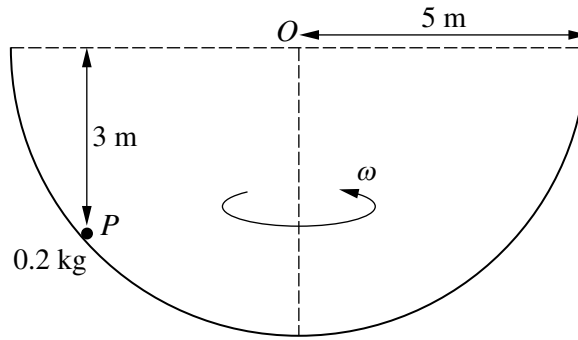
7

Fig. 1

A particle *P* of mass 0.2 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre *O* and radius 5 m. *P* moves with constant angular speed ω in a horizontal circle at a vertical distance of 3 m below the level of *O* (see Fig. 1).

  (i) Calculate the magnitude of the force exerted by the hemisphere on *P*. **[3]**
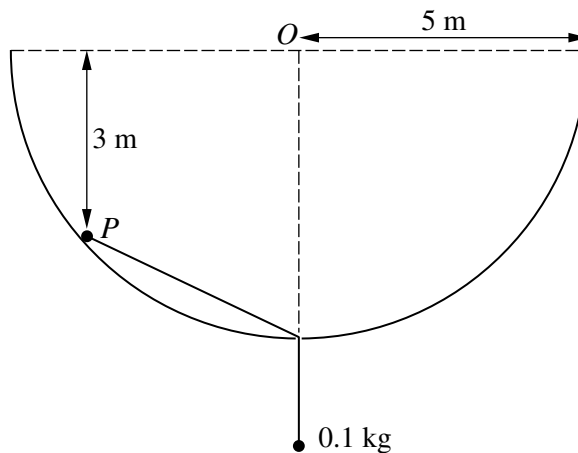
  (ii) Calculate ω. **[4]**

Fig. 2

A light inextensible string is now attached to *P*. The string passes through a small smooth hole at the lowest point of the hemisphere and a particle of mass 0.1 kg hangs in equilibrium at the end of the string. *P* moves in the same horizontal circle as before (see Fig. 2).

  (iii) Calculate the new angular speed of *P*. **[8]**

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1	$75 \times 9.8 \times 40$	B1	Average Speed = $40 \div 120$
	$(75 \times 9.8 \times 40) \div 120$	M1	$(75 \times 9.8) \times$ (Average speed)
	245 W	A1 **[3]**	**3**

2 (i)	$v^2 = 2 \times 9.8 \times 3$ or $2 \times 9.8 \times 1.8$	M1	Kinematics or energy
	$v_1 = \sqrt{6g}$ or $\sqrt{58.8}$ or $\frac{7}{5}\sqrt{30}$ or 7.67	A1	Speed of impact ($\pm$)
	$v_2 = \sqrt{3.6g}$ or $\sqrt{35.28}$ or $\frac{21}{5}\sqrt{2}$ or 5.94	A1	Speed of rebound ($\pm$)
	$I = \pm\, 0.2(5.94 + 7.67)$	M1	
	2.72	A1ft **[5]**	+ve, ft on v_1 and v_2
(ii)	$e = 5.94/7.67$	M1	
	0.775 or $\frac{\sqrt{15}}{5}$	A1ft **[2]**	Allow 0.774, ft on v_1 and v_2 **7**

3 (i)	$\bar{u} = 0.2$ (from vertex) or 0.8 or 0.1	B1	com of conical shell
	$0.5\bar{d} = 0.2 \times \bar{u} + 0.3 \times 0.65$	M1	
		A1	
	$\bar{d} = 0.47$	A1 **[4]**	**AG**
(ii)	$s = 0.5$	B1	slant height, may be implied
	$T\sin 80° \times 0.5 = 0.47 \times 0.5 \times 9.8$	M1	
		A1	
	$T = 4.68$ N	A1 **[4]**	**8**

4 (i)	$D - 400 = 700 \times 0.5$	M1	3 terms
	$D = 750$ N	A1 **[2]**	
(ii)	$P = 750 \times 12$	M1	
	9 000 W or 9 kW	A1ft **[2]**	
(iii)	$P/35 = 400$	M1	
	14 000 W or 14 kW	A1 **[2]**	
(iv)	$D = 14000/12$	B1ft	May be implied
	$3500/3 = 400 + 700 \times 9.8\sin\theta$	M1	3 terms
		A1	Their P/12
	$\theta = 6.42°$	A1 **[4]**	**10**

5	$16 - 12 = 2x + 3y$	M1	
	$4 = 2x + 3y$	A1	aef
	$½.2(8)^2 + ½.3(4)^2$ or $½.2x^2 + ½..3y^2$ or $±½.2(8^2 - x^2)$ or $±½.3(4^2 - y^2)$	B1	
	$½.2(8)^2 + ½.3(4)^2 - ½.2x^2 - ½...3y^2 = 81$	M1	
	$2x^2 + 3y^2 = 14$	A1	aef
	Attempt to eliminate x or y from a linear and a quadratic equation	M1	
	$15y^2 - 24y - 12 = 0$ or $10x^2 - 16x - 26 = 0$	A1	aef
	Attempt to solve a three term quadratic	M1	
	$x = -1$ (or $x = 2.6$)	A1	
	$y = 2$ (or $y = -2/5$)	A1	
	$x = -1$ and $y = 2$ only	A1	
	speeds 1, 2 away from each other	A1 [12]	**12**
6 (i)	$30^2 = V_1^2 \sin^2 \theta_1 - 2×9.8×250$	M1	$½m\, V_1^2 = ½m\, 50^2 + m×9.8×250$
	$V_1^2 \sin^2 \theta_1 = 5800$ AEF	A1	
	$V_1 \cos \theta_1 = 40$	B1	
	$V_1 = 86.0$	A1	**AG**
	$\theta_1 = 62.3°$	A1 [5]	**AG**
(ii)	$0 = \sqrt{5800}\, t_p - 4.9t_p^2$	M1	$30 = V_1 \sin \theta_1 - 9.8t$
	$t_p = 15.5$	A1	
			$t = 4.71$
	$-\sqrt{5800} = 30 - 9.8t_q$	M1	
	$t_q = 10.8$	A1 [4]	
(iii)	$R = 40×15.5$	M1	
	$R = 621$	A1	(620, 622)
	$V_2 \cos \theta_2 × 10.8 = 621$	B1	$V_2 \cos \theta_2 = 57.4$
	$0 = V_2 \sin \theta_2 × 10.8 - 4.9 × 10.8^2$	M1	
	$V_2 \sin \theta_2 = 53.1$ or 53.0	A1	(52.9,53.1)
	Method to find a value of V_2 or θ_2	M1	
	$\theta_2 = 42.8°$	A1	42.6° to 42.9°
	$V_2 = 78.2$ m s^{-1} or 78.1 m s^{-1}	A1 [8]	or 78.1° **17**
7 (i)	$\cos\theta = 3/5$ or $\sin\theta = 4/5$ or $\tan\theta = 4/3$ or $\theta = 53.1°$	B1	θ = angle to vertical
	$R\cos\theta = 0.2×9.8$	M1	
	$R = 3.27$ N or $49/15$	A1 [3]	
(ii)	$r = 4$	B1	
	$R\sin\theta = 0.2×4×\omega^2$	M1	
		A1	
	$\omega = 1.81$ rad s^{-1}	A1 [4]	

(iii)	$\varphi = 26.6°$ or $\sin\varphi = \dfrac{1}{\sqrt{5}}$ or $\cos\varphi = \dfrac{2}{\sqrt{5}}$ or	B1	φ = angle to horizontal
	$\tan\varphi = 0.5$		
	$T = 0.98$ or $0.1g$	B1	
	$N\cos\theta = T\sin\varphi + 0.2\times9.8$	M1	Vertically, 3 terms
	$N\times3/5 = 0.438 + 1.96$	A1	
	$N = 4.00$	A1	may be implied
	$N\sin\theta + T\cos\varphi = 0.2\times4\times\omega^2$	M1	Horizontally, 3 terms
	$4\times4/5 + 0.98\cos 26.6° = 0.8\omega^2$	A1	
	$\omega = 2.26$ rad s^{-1}	A1 **[8]**	**15**